State space ARIMA for supply chain forecasting

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SMUG2019

16th April 2019

Marketing Analytics and Forecasting

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- Teaching, including executive for companies;
- Consulting;
- Research projects;
- Developing software.

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An exciting story of a mathematical model...

Introduction

ARIMA is a popular model in academic world.

It originates from Box and Jenkins (1976).

Did you know that Gwilym Jenkins worked at Lancaster University?

Anyway, ARIMA is popular in:

- time series analysis,
- economics,
- finance,
- engineering,
- ...

Introduction

ARIMA is used for theoretical derivations in supply chain (Kim et al., 2003; Wang et al., 2010; Ali and Boylan, 2012; Syntetos et al., 2016, etc.)

But it is not used widely in business.

Several papers have shown over the years that the most popular forecasting method is simple moving average and / or simple exponential smoothing (Winklhofer et al., 1996; Weller and Crone, 2012).

ARIMA allows capturing dependencies that these two cannot.

Is something wrong with ARIMA?

Introduction

It seems that the main problem is the length of history.

It is very common for companies to preserve at most 3 last years of data.

This is a small sample for ARIMA:

- Applying seasonal ARIMA becomes difficult (loss of observations);
- Parametric statistical tests might be not powerful;
- The order selection mechanism might be computationally expensive.

And also it's complicated...

We need to save the world!

State space ARIMA

The general Seasonal ARIMA has the following simple form:

$$\phi_p(B)\delta_d(B)\Phi_P(B^m)\Delta_D(B^m)y_t = \theta_q(B)\Theta_Q(B^m)\epsilon_t + \beta \qquad (1)$$

Anyway, let's move on and never return to this again!

This model can be represented in a simpler form of Snyder (1985).

It's called single source of error (SSOE) state space model.

State space ARIMA

The advantages of SSARIMA:

- The initialisation can be done in a separate initial set;
- This means that we don't loose observations;
- Different orders can be selected without data transformations;
- No need for statistical tests, selection can be done using information criteria.

State space ARIMA

Still, order selection might take time.

e.g. if the maximum orders of the model correspond to SARIMA(3,2,3)(2,1,2)$_m$, we need to check 1728 models for each time series.

Not doable in finite time!

Svetunkov & Boylan (2019) developed an algorithm that reduces the number of models to check to up to 31.

This algorithm was adapted for Smoothie, reducing the pool even further.

State space ARIMA

An experiment on supply chain data (4267 SKUs) showed that the approach is robust and performs well.

Model	MPE	MAPE	ARMAE
ARIMA	-4.1	33.5	97.4
SSARIMA	*-1.9*	*32.5*	*92.9*
SSARIMA Ext	**-0.5**	35.0	95.8
Smoothie ARIMA	2.3	**30.0**	**82.4**

Table: Median error measures (percentages).

Smoothie ARIMA is 17.6% more accurate than Naive.

State space ARIMA

And it is quite fast...

Model	Time in minutes
Conventional ARIMA	39.82
SSARIMA	47.86
SSARIMA Ext	17,989.72
Smoothie ARIMA	27.36

Table: Time of computation for each model in minutes for all 4267 series in **serial**.

What have we done?!

Conclusions

- ARIMA is flexible model,
- It is popular in academia,
- But there are limitations of applying it in supply chain context,
- We used an exiting approach to reformulate it,
- We developed an efficient algorithm for model selection and estimation,
- It is fast and accurate,
- Now you can enjoy ARIMA in Smoothie!

Thank you for your attention!

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