Why you should care about exponential smoothing

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Introduction

Exponential smoothing (ES) is one of the most popular methods in demand forecasting practice:

Weller and Crone (2012) report that ES was used the most frequently by practitioners, in ∼32% of cases.

Any major statistical software has it and relies on it (Fildes et al., 2018; Fildes, 2020; Schaer et al., 2022).

Introduction

Yet some machine learning experts seem to have strong views about exponential smoothing and forecasting:

"Applied forecasting academia hasn't created anything useful over the last 40 years"

"Exponential smoothing is just a special case of ARIMA"

"Exponential smoothing is the model from the black and white era TV"

"The main disadvantage of ES ... is its inability to correctly factor in and handle market shifts and trends"

Introduction

I feel that there are many misconceptions about ES.

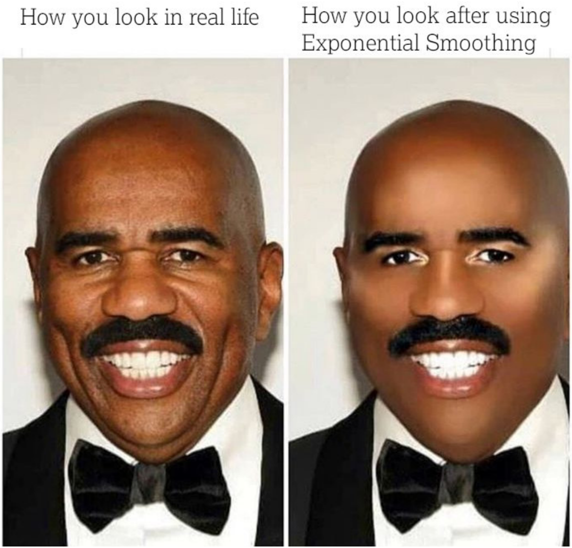
And a large misunderstanding around it.

I decided to make a presentation to clarify this.

DISCLAIMER. some of the models are discussed in my monograph:

Forecasting and Analytics with the Augmented Dynamic Adaptive Model (ADAM)

Basics of ES


How you look in real life
How you look after using Exponential Smoothing

Basics of ES

Good overview of the past of ES is done by Gardner (1985) and Gardner (2006).

Here we discuss the main ES methods.

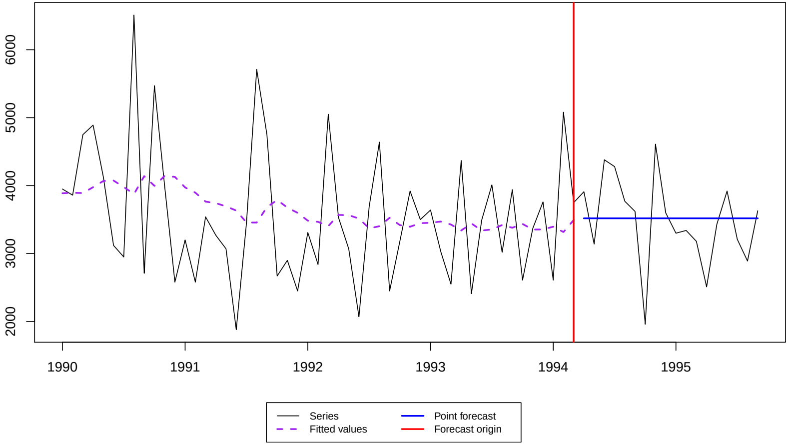
Simple Exponential Smoothing (SES) method was proposed by Brown (1956) and Holt (2004) in the form:

$$\hat{y}_{t+1} = \hat{\alpha} y_t + (1 - \hat{\alpha}) \hat{y}_t, \tag{1}$$

whre y_t is the actual, $\hat{y}_t$ is the predicted value, $\hat{\alpha}$ is the smoothing parameter.

SES has an underlying ARIMA(0,1,1) model (Muth, 1960).

Simple Exponential Smoothing



Holt's method

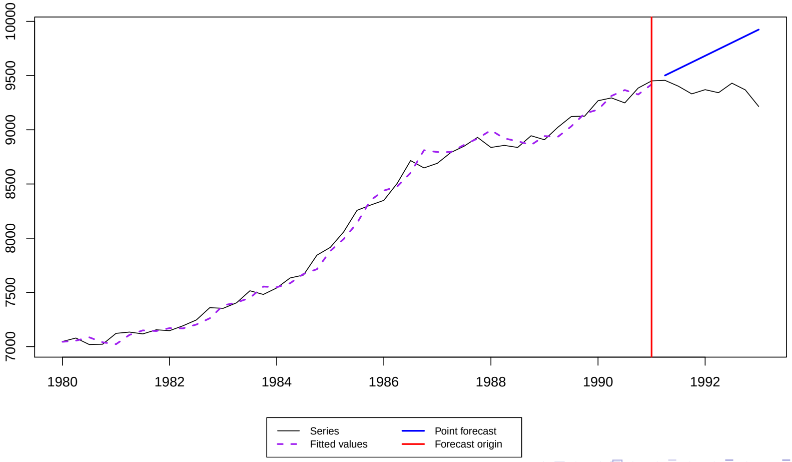
Holt (2004) proposed the trend model in 1957 (presented here in error correction form):

$$\begin{aligned} \hat{y}_{t+1} &= \hat{l}_t + \hat{b}_t \\ \hat{l}_t &= \hat{l}_{t-1} + \hat{b}_{t-1} + \hat{\alpha} e_t, \\ \hat{b}_t &= \hat{b}_{t-1} + \hat{\beta} e_t \end{aligned} \tag{2}$$

where $\hat{l}_t$ is the level of series, $\hat{b}_t$ is the trend component and $\hat{\beta}$ is the smoothing parameter.

Nerlove and Wage (1964) demonstrated that this has an underlying ARIMA(0,2,2) model.

Holt’s method



Holt-Winters method

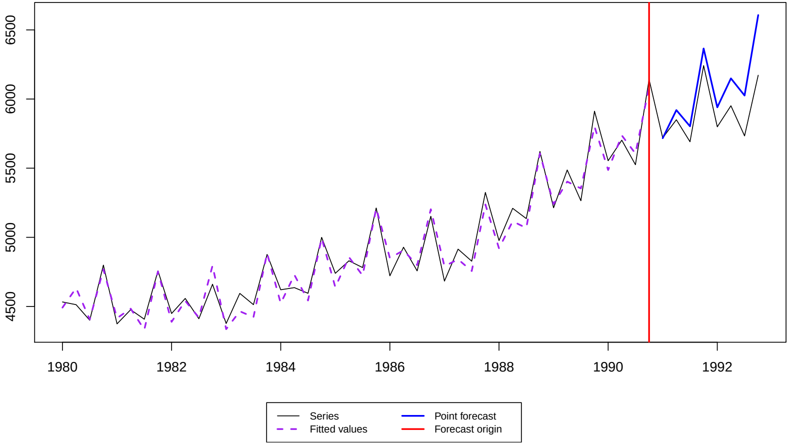
Winters (1960) developed a seasonal version of Holt's method that contained seasonal indices:

$$\begin{array}{l} \hat{y}_{t+1} = (\hat{l}_t + \hat{b}_t)\hat{s}_{t-m+1} \\ \hat{l}_t = \hat{l}_{t-1} + \hat{b}_{t-1} + \hat{\alpha}\frac{e_t}{\hat{s}_{t-m}} \\ \hat{b}_t = \hat{b}_{t-1} + \hat{\beta}\frac{e_t}{\hat{s}_{t-m}} \\ \hat{s}_t = \hat{s}_{t-m} + \hat{\gamma}\frac{e_t}{\hat{s}_{t-m}} \end{array}, \quad (3)$$

where $\hat{s}_t$ is the seasonal component.

Chatfield (1977) showed that there is no underlying ARIMA for the multiplicative seasonal Holt-Winters method (3).

Holt-Winters method



ARIMA vs ES

Some ES models have underlying ARIMA...

...so, many thought that ARIMA was better than ETS.

Makridakis et al. (1982) conducted an independent experiment.

ARIMA was constructed based on BJ methodology (Box and Jenkins, 1976).

ES and ARARMA performed better than the others.

Statistical community did not like the results...

Damped trend

Roberts (1982) proposed a model that was then picked up by Gardner and McKenzie (1985), expanding the set of ES methods:

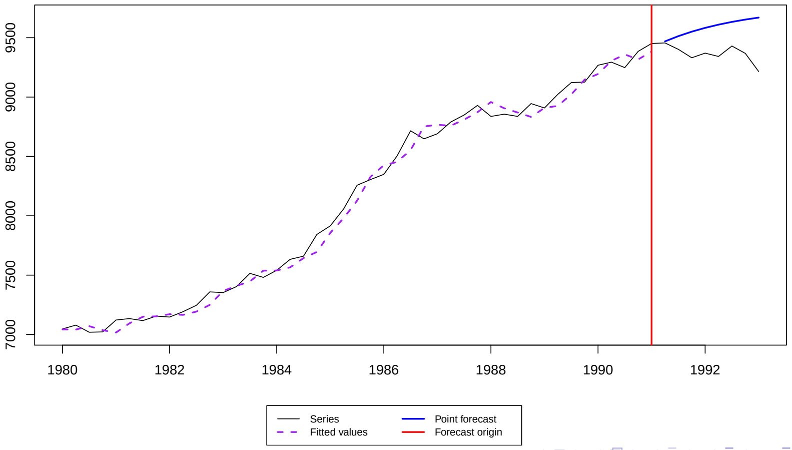
$$\begin{aligned}
&\hat{y}_{t+1} = \hat{l}_t + \hat{\phi}\hat{b}_t \\
&\hat{l}_t = \hat{l}_{t-1} + \hat{\phi}\hat{b}_{t-1} + \hat{\alpha} e_t, \\
&\hat{b}_t = \hat{\phi}\hat{b}_{t-1} + \hat{\beta} e_t
\end{aligned} \tag{4}$$

where $\hat{\phi} \in (0,1)$ is a damping parameter.

This method has an underlying ARIMA(1,1,2) model.

Gardner and McKenzie (1989) discuss seasonal version of this.

Damped trend (Gardner's method)



SSOE state space model

Snyder (1985) modified MSOE and proposed a "Single Source of Error" state space model:

$$\begin{aligned} y_t &= \mathbf{w}'\mathbf{v}_{t-1} + \epsilon_t \\ \mathbf{v}_t &= \mathbf{F}\mathbf{v}_{t-1} + \mathbf{g}\epsilon_t \end{aligned}, \qquad (5)$$

where $\mathbf{g}$ is a persistence vector, containing smoothing parameters.

Now all the components are influenced by the same error.

This is much easier to understand and estimate than MSOE.

SSOE state space model

An example is a local level model, where $\mathbf{w} = 1$, $\mathbf{F} = 1$ and $\mathbf{g} = \alpha$:

$$\begin{aligned} y_t &= l_{t-1} + \epsilon_t \\ l_t &= l_{t-1} + \alpha \epsilon_t \end{aligned} . \tag{6}$$

It underlies SES.

ARIMA vs ES, part 2

In the M3 competition (Makridakis and Hibon, 2000), automatic ARIMA performed worse than Damped trend.

SSOE ES did not participate.

ES methods did well, but did not win.

Theta won.

Theta relies on SES.

SSOE state space model

Ord et al. (1997) expanded SSOE for the cases of multiplicative components and multiplicative error:

$$\begin{aligned} y_t &= w(\mathbf{v}_{t-1}) + r(\mathbf{v}_{t-1})\epsilon_t \\ \mathbf{v}_t &= f(\mathbf{v}_{t-1}) + g(\mathbf{v}_{t-1})\epsilon_t \end{aligned}, \quad (7)$$

so that SSOE now underlied all the possible ES methods.

Hyndman et al. (2002) expanded the Pegels (1969) taxonomy.

Given 2 types of errors, 5 types of trends and 3 types of seasonal components, SSOE has 30 models.

SSOE state space model

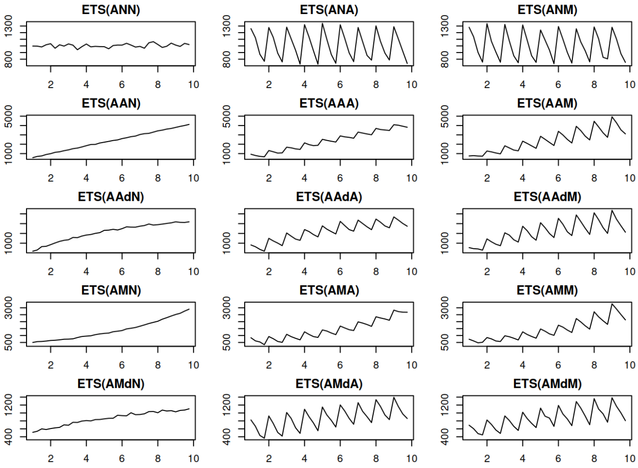
Hyndman et al. (2008) called this framework ETS – Error, Trend, Seasonality.

ETS is relatively easy to use and estimate, it provides appropriate prediction intervals and supports model selection (with information criteria).

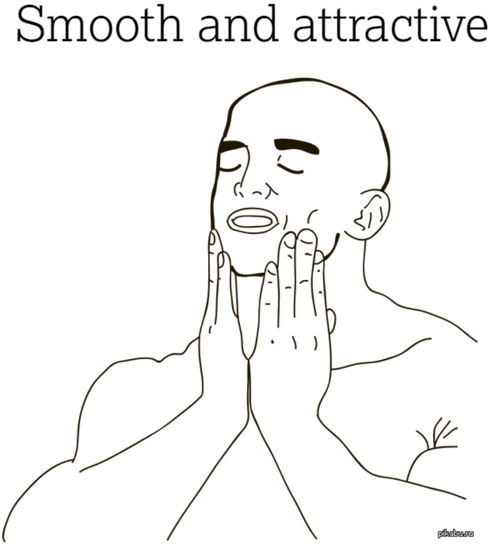
It underlies 15 ES methods, including methods by Pegels (1969) and Taylor (2003).

It can be extended by inclusion of exogenous variables, additional seasonal components or other variables.

ETS framework



Why is Exponential Smoothing still attractive?



How can one make a method trustworthy?

Spavound and Kourentzes (2022) discuss four factors impacting the trustworthiness:

1. Reliability (does it perform consistently across series);
2. Stability (does it perform consistently across time);
3. Intelligibility (explainability of key elements);
4. Alignment (align with objective).

Also see Simon's presentation:
https://www.youtube.com/watch?v=EfHpaSFAtlc

1. Reliability

ES is reported to perform well in many competitions (Makridakis et al., 1982; Fildes et al., 1998; Makridakis and Hibon, 2000; Athanasopoulos et al., 2011; Makridakis et al., 2020).

It does not necessarily come first, but it is hard to break.

Makridakis et al. (2022) showed that 92.5% of submissions for M5 failed to outperformed bottom-up ES.

Kolassa (2020) showed that the winner of M5 competition outperformed bottom-up ES on 58.5% of series in terms of MSE.

The second-place method outperformed bottom-up ES only on 6.7% of series.

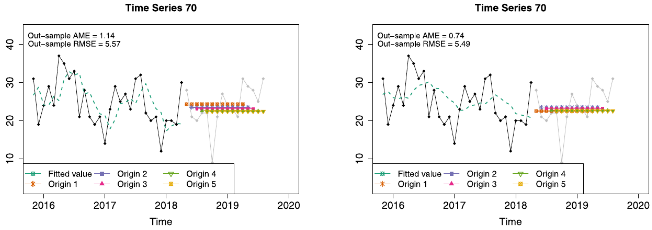
So, it is a robust forecasting model.

2. Stability

In ES, stability comes from the smoothing parameters.

If they are too high, the forecast becomes unstable.

Pritularga et al. (2023) showed how regularisation can hep ETS.



(a) MLE, $\hat{\alpha}_{\text{MLE}} = 0.47$ (b) Shrinkage, $\hat{\alpha}_{\text{REG}} = 0.19$

Fig. 1. Examples of different estimation approaches for five origins. The AME is the absolute mean error, and both error measures calculate the accuracy of one- to 12-step-ahead forecasts.

3. Intelligibility

Many demand planners do not know forecasting.

A lot of them do not even know statistics.

Explain this (SARIMA(1,1,2)(0,1,0)$_4$):

$$y_t(1-\phi_1 B)(1-B)(1-B^4) = \epsilon_t(1+\theta_1 B+\theta_2 B^2)$$

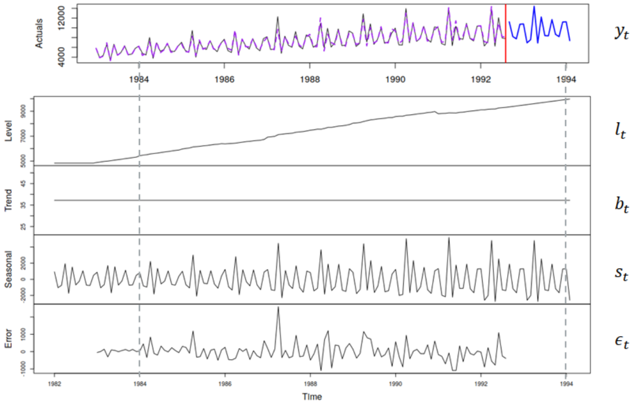
or this:

$$\hat{y}_{t+1} = \hat{\alpha} y_t + (1-\hat{\alpha})\hat{y}_t$$

Which one is simpler?

3. Intelligibility

$$y_t = l_{t-1} + b_{t-1} + s_{t-m} + \epsilon_t$$



4. Alignment

This can be done by aligning the forecasts from ETS with decisions:

1. Produce cumulative over the lead time forecasts;
2. Generate prediction intervals/specific quantiles;
   - Svetunkov (2023a) discusses in Chapter 18 different options for point forecasts, quantiles and intervals.

+ using loss functions, aligning with decisions (Kourentzes et al., 2019; Saoud et al., 2022; Svetunkov et al., 2023).

Is it accurate?

So, ES ticks all four boxes without much trouble.

But we do not expect it to perform always great.

Nixtla implemented ETS in StatsForecast library.

It works fine.

It does not always outperform ML methods (why would it?)

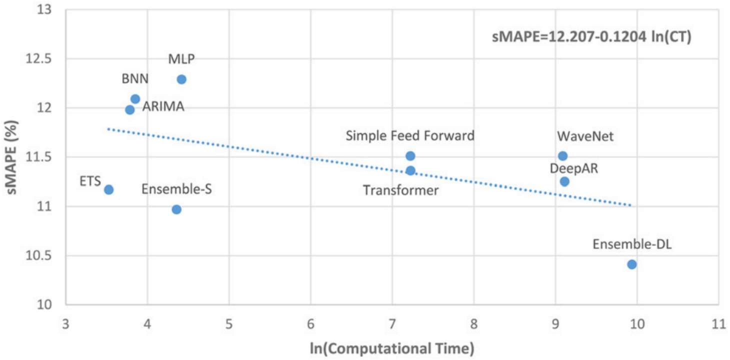
Is it accurate?

Table 1: Main performance results for TimeGPT with zero-shot inference and benchmark models measured with rMAE and rRMSE, lower scores are better. The best model for each frequency and metric is highlighted in bold, the second best underlined, and the third best underlined with a dotted line.

	Monthly		Weekly		Daily		Hourly	
	rMAE	rRMSE	rMAE	rRMSE	rMAE	rRMSE	rMAE	rRMSE
ZeroModel	2.045	1.568	6.075	6.075	2.989	2.395	10.255	8.183
HistoricAverage	1.349	1.106	4.188	4.188	2.509	2.057	2.216	1.964
SeasonalNaive	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Theta	0.839	0.764	1.061	1.061	0.841	0.811	1.163	1.175
DOTheta	0.799	0.734	1.056	1.056	0.837	0.806	1.157	1.169
ETS	0.942	0.960	1.079	1.079	0.944	0.970	0.998	1.009
CES	1.024	0.946	1.002	1.002	0.919	0.899	0.878	0.896
ADIDA	0.852	0.769	1.364	1.364	0.908	0.868	2.307	2.207
IMAPA	0.852	0.769	1.364	1.364	0.908	0.868	2.307	2.207
CrostonClassic	0.989	0.857	1.805	1.805	0.995	0.933	2.157	2.043
LGBM	1.050	0.913	0.993	0.993	2.506	2.054	**0.733**	**0.709**
LSTM	0.836	0.778	1.002	1.002	0.852	0.832	0.974	0.955
DeepAR	0.988	0.878	0.987	0.987	0.853	0.826	1.028	1.028
TFT	0.752	0.700	0.954	0.954	0.817	0.791	1.120	1.112
NHITS	0.738	0.694	0.883	0.883	**0.788**	**0.771**	0.829	0.860
TimeGPT	**0.727**	**0.685**	**0.878**	**0.878**	0.804	0.780	0.852	0.878

https://doi.org/10.48550/arXiv.2310.03589

Is it fast?



Makridakis et al. (2023)

Can it be improved?

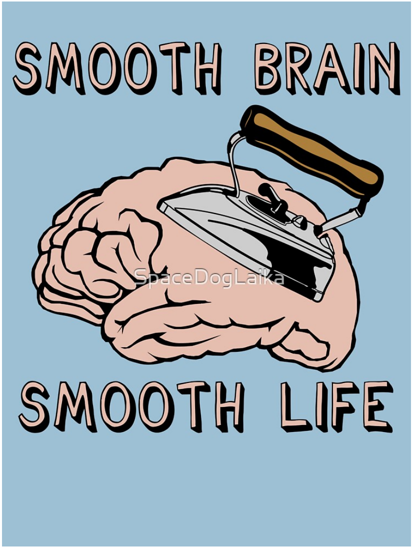
The conventional model was developed for regular demand.

What about...

- explanatory variables,
- intermittent demand,
- and/or multiple frequencies?

Svetunkov (2023a) develops these and other aspects of ETS.

Can ES be improved?



Explanatory variables, ETSX

ETSX is first mentioned in Chapter 9 of Hyndman et al. (2008).

It was then used by Koehler et al. (2012) for outliers handling.

Kourentzes and Petropoulos (2016), Ramos et al. (2023) used ETSX in their studies.

It outperformed the one without X.

For Abolghasemi et al. (2020), ETSX performed poorly.

ETSX is implemented in the `adam()` function for the `smooth` package (Svetunkov, 2023b) in R (R Core Team, 2023).

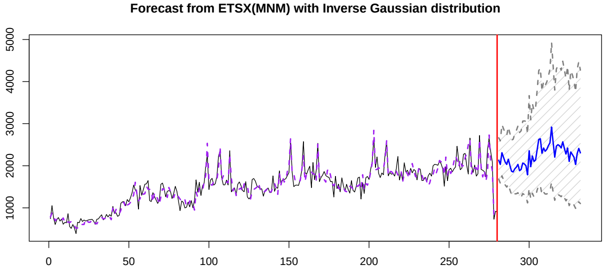
See Chapter 10 of Svetunkov (2023a).

ETSX in R

ETSX(M,N,M) with dummy for promotions, outliers and their lags:

```
xreg <- data.frame(y, xregExpander(x, lags=-c(1,2), gaps="zero"))

adamModel <- adam(xreg, "MNM", lags=c(1,52), h=52, holdout=FALSE)
```



Intermittent demand, iETS

Svetunkov and Boylan (2023) proposed an intermittent state-space model based on the pure multiplicative ETS.

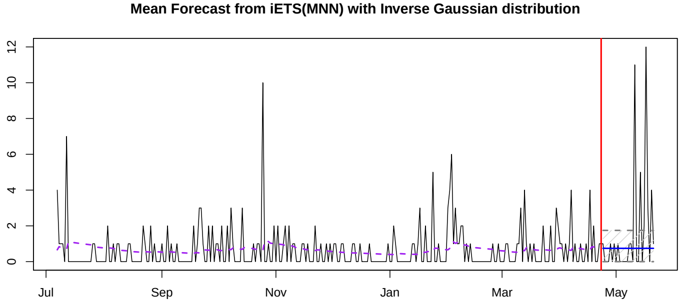
$$y_t = o_t z_t, \tag{8}$$

- It extends the ETS taxonomy,
- allows capturing trends and seasonality,
- allows using exogenous variables for both demand sizes and demand occurrence.

Implemented in adam(). See Chapter 13 of Svetunkov (2023a).

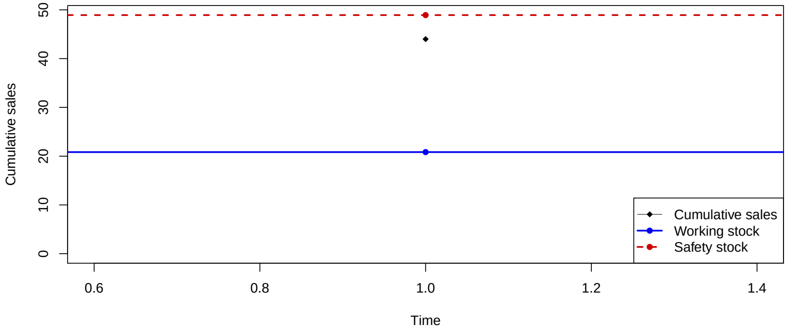
iETS in R

```
adaModel <- adam(y, "YYY", occurrence="direct", h=28,
holdout=TRUE)
forecast(adaModel, h=28, interval="prediction",
nsim=10000, cumulative=TRUE, side="upper") |> plot()
```



iETS in R

Compare cumulative sales over the 28 days with the forecasts:



Demand with multiple seasonalities

Taylor (2010) proposes triple-seasonal Holt-Winters, using the same principle as the double-seasonal one.

Taylor and Snyder (2012) propose parsimonious seasonal exponential smoothing, using the same seasonal parameters for some of periods.

Demand with multiple seasonalities

De Livera (2010) proposes BATS model and De Livera et al. (2011) extend it to TBATS.

TBATS stands for Trigonometric, Box–Cox transform, ARMA errors, Trend, and Seasonal components.

TBATS$(\omega, \phi, p, q, \{m_1, k_1\}, \{m_2, k_2\}, ..., \{m_T, k_T\})$

BATS is a generalisation of ETS (without "E"), double- and triple-seasonal Holt-Winters.

TBATS solves the problem with fractional seasonality via Fourier transform.

Demand with multiple seasonalities

adam() supports multiple seasonal ETS/ARIMA, see Chapter 12 of Svetunkov (2023a).

The default mechanism is similar to Taylor (2010).

But it supports all ETS models.

And it can work fast if backcasting is used for initialisation.

Demand with multiple seasonalities

ETS(M,N,M)$_{48,336}$ + AR(1):

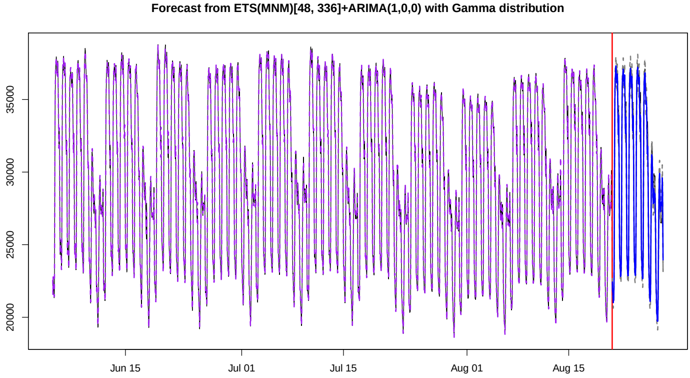
```
adamETSMNMAR <- adam(y, "MNM", lags=c(1,48,336), initial="back",
orders=c(1,0,0), h=336, holdout=TRUE, maxeval=1000)
```

Takes ∼2 seconds to evaluate on 3696 observations.

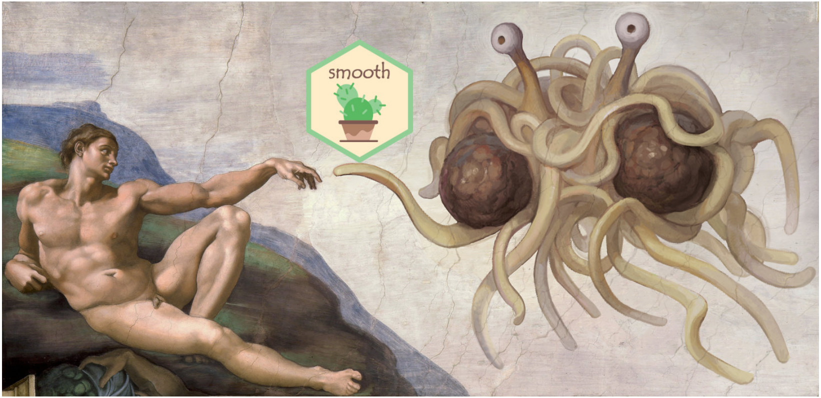
You can add Fourier terms and update them in ADAM ETSX to get something similar to TBATS.

Demand with multiple seasonalities

```
forecast(adamETSMNMAR, h=336, interval="pred") |> plot()
```



Conclusions



Conclusions

- ES has not been a "model from the black & white TV era" for a long time;
- It has evolved constantly since 1956;
- Huge progress in this area for the last 40 years;
- It is not a special case of ARIMA;
- In fact, you can make them work together;
- Handle external information with ETS;
- Deal with intermittent demand and/or multiple frequencies;
- ADAM presents a modern view on ETS and ARIMA.

Conclusions

If you see this ES formulation, run away!

Level:

$$l_t = (1-\alpha)l_{t-1} + \alpha x_t$$

Trend:

$$b_t = (1-\beta)b_{t-1} + \beta(l_t - l_{t-1})$$

Seasonal:

$$c_t = (1-\gamma)c_{t-L} + \gamma(x_t - l_{t-1} - b_{t-1})$$

Model:

$$y_t = (l_t + b_t)c_t$$

Forecasting:

$$\hat{y}_{t+n} = (l_t + nb_t)c_{t-L+1+(n-1) mod L}$$

Conclusions

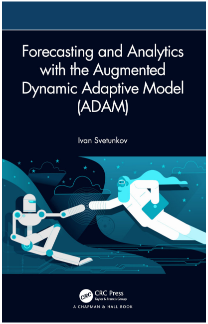
You should use state space!

$$
\begin{aligned}
y_t &= l_{t-1} + b_{t-1} + s_{t-m} + \epsilon_t \\
l_t &= l_{t-1} + b_{t-1} + \alpha\epsilon_t \\
b_t &= b_{t-1} + \beta\epsilon_t \\
s_t &= s_{t-m} + \gamma\epsilon_t
\end{aligned} .
$$

- If you use the old ES formulation, then you are stuck in 50s – 60s;
- Get unstuck! Use modern ETS!

Thank you for your attention!

Ivan Svetunkov



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